Developing student problem solving skills in mathematics and engineering

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My background: problem solving course development and implementation

Developed a **dedicated problem solving course** for first year engineering students. Approximately 70 – 80 students. Lectures run as workshops, working on skills. Weekly assessable lab sessions. Based upon the McMaster Problem Solving Program.

**Applied to a second-year engineering mathematics course.** Used many of the techniques and ideas in this mathematics course. Approximately 100 students. This was run in flipped mode. Course material online. In lectures students work through worksheets and discuss concepts.

Typically around 90% lecture attendance.

Students from all over Australia. Some international. Clear career path on graduation.
Motivation behind the problem solving course: anecdotal

One of the primary skills employers look for in graduates is problem-solving ability.

Lecturers complain about poor student problem-solving ability.

Traditional approach: Give lots of problems, show how to solve them. Emphasis on doing.

Result: Students seem to show little, if any, improvement in problem solving ability, and are easily thrown by unfamiliar problems.
Motivation behind the problem solving course: evidence

One academic at McMaster University enrolled in the engineering program and attended all classes of the four year program. Found: Students solve 3013 problems in 4 years
- solve 45 open-ended problems
- watch lecturers solve 1000 problems
- work problems on board

Students showed no improvement in problem solving confidence over the 4 years.

For details of the study see:
Conclusion...

To see improvement in problem-solving, the skills must be taught and practiced explicitly.

**Difficulty**: problem-solving is a hidden skill.

To improve at golf you can watch an expert on youtube, or have a coach correct your swing. But problem solving goes on inside your head, and it’s hard to see what’s happening in there.

**Solution**? To begin with, we have to make problem solving somehow ‘visible’ so we can set about improving it.
Following the McMaster Problem Solving Program: course design

Emphasis on student activity in lectures (‘workshop mode’) and labs (assessable activities, including problem based learning).

For each course area:

Define skills
Motivate use & importance
Provide Pre-test, Objectives, Targets
Outline route ahead

**Build** skill: activity + feedback + reflect
Students gather own research evidence

**Bridge** skill: activity + feedback + reflect
Post-test, Objectives
DISCOVERY (more detailed reflection)

Include in reflective journal and **Extend** to everyday problems (i.e. Application of learning outside course)
Student Evaluation

Positives:
“The problem solving techniques taught as they would be with us for the rest of our lives”
“It supplemented our other courses very well with both general problem solving and course specific examples.”
“Labs and lectures were very active and dynamic”
“Can use the knowledge gained here in other courses”
“Really helped me develop my problem solving ability”
“Learning to become more methodical when solving problems”
“The journal”

Suggestions:
“Make it harder”
“No final exam”
“More guidance on assessment early on”
“Exploration into relevance for engineering (apart from how to solve problems)”
Some modifications in this year’s iteration (to run in semester 2)

Classes are flipped. All material online. **No labs or tutorials.** Assessable lab-like activities will take place in lectures.

Assessment will be conducted in a variety of modes, including online quizzes, self-assessed and peer-assessed activities, in addition to more traditional lecturer-moderated activities.

Emphasis will be on practically building the skills using scaffolded problems, in mathematics, programming, context-free (puzzles), and engineering.

More development of the psychology underlying thinking and problem solving, making explicit the processes involved.

**Goal:** To see better transfer of skills to specific domains, such as mathematics.
Let’s start by defining problems

An **exercise** is a situation that, by pattern recognition, the person recognizes as being similar to a problem that he/she remembers solving successfully in the past. To solve the new situation, the persons recalls and applies the past solution. Of 100 “problem situations” encountered by persons with about 10 years experience in their specialty over 90 of them are *exercises*. They rarely encounter “problems”.

A **problem** is a situation where the person cannot recall any past solved situation that bears any resemblance to the situation. They are unsure or what the problem really is and of how to tackle it. They make a lot of mistakes. They try many different options and “see what happens”. They become distressed. They need to brainstorm many ideas. They try simpler versions of the problem.
Analysing our own thinking first…what goes on when we answer questions?

1. 

   24 
   x 63 

   Recognised this as a multiplication problem, some mental effort involved, but clear path to solution

2. 

   Notice anything special about these two?

   Not clear what this is about, but couldn’t help reading it.

Implications
Some activities require dedicated mental effort, others we can do seemingly without thinking. When we do things without thinking we may make choices without even being aware of it.

Questions taken from Kahneman, 2011
Lessons from psychology
(Kahneman, *Thinking, fast and slow*, 2011)

Our thinking can be divided into two types:

**System 1 (fast)** operates automatically and quickly, with little or no effort and no sense of voluntary control.

**System 2 (slow)** allocates attention to the effortful mental activities that demand it, including complex computations. The operations of System 2 are often associated with the subjective experience of agency, choice, and concentration.

System 1 has inherent biases, and affects decisions made by system 2. System 2 requires mental energy to maintain, and often only gets called on when faced with something system 1 can’t solve.
Some more questions to try…

If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?

A bat and ball cost $1.10. The bat costs one dollar more than the ball. How much does the ball cost?

Both questions have a simple, intuitive, wrong answer.

The fact that the answer is wrong can be easily checked, but requires mobilisation of system 2.

These sorts of questions are used as part of a test to see the tendency of students to answer questions using system 1 or system 2 thinking.

Kahneman, 2011
Law of least effort

**Law of least effort.** People gravitate to the least demanding course of action.

We **normally seek to avoid mental overload** by dividing our tasks into multiple easy steps.

We like to go at a mental walking pace.

**Self-control requires attention and effort.** If you have had to force yourself to do something, you are less willing or less able to exert self-control when the next challenge comes around.

**Students** who don’t have the tools to reduce mental load, or who try to avoid mental load, may:
- Give up.

93ih.wordpress.com
So, what sort of effort is required?

What is involved when solving problems?
What we know about good problem solvers (Whimbey)

1. **Positive Attitude**: strong belief that academic reasoning problems can be solved through careful, persistent analysis.

2. **Concern for Accuracy**: take great care to understand the facts and relationships in a problem fully and accurately. Almost compulsive in checking if understanding is correct and complete.

3. **Break the Problem into Parts**: tackle piece by piece.

4. **Avoid Guessing**: work methodically, carefully, no jumps.

5. **Active in Problem Solving**: do things to try to understand and answer difficult questions (e.g. reword, draw diagrams, ‘talk to themselves’).

All require the student to slow down, and have a belief that they can solve the problem by working slowly through a problem using “system 2”, using techniques to reduce cognitive overload.
What aspect has the largest impact on capability as a problem solver?

1. Attitude
2. Knowledge
3. Strategy
4. Experience
5. Creativity
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If students are intrinsically motivated (rather than extrinsically through chasing a reward for instance) then many challenges of the teaching and learning environment go away.

Ideally the student should:
- Have belief in success of their skills and process.
- Be motivated by learning goals (rather than performance goals).
- Be problem-finders rather than problem-followers.
What do we get students to do?

Whether a situation is a problem or an exercise depends on the person and his/her experience.

Typically, most “problems” we give to students are really exercises (i.e. they encourage pattern recognition, rather than problem solving skills).

This makes it hard for students to develop problem solving skills, and can end up fostering a counter-productive attitude (e.g. belief that you don’t need to think, and in exams it’s a matter of luck, either having done the question before, or seeing the “trick”).
A start point...exhortations to students

From McMaster Problem Solving Strategy, Step 1.
“I want to and I can”

But, I think all the motivation theory and thinking theory comes down to two things...

1. “Use your brain”
2. “Don’t be afraid to make mistakes”
Building problem solving skills…Step 1: Attitude

Good problem solvers have confidence in their ability and are not defeated at the first hurdle.

Students need to develop problem solving confidence.

**Tip:** Try the Build-Bridge-Extend approach of the McMaster program.

**Build:** Build the skill in a stress-free exercise so that students can focus on the mental processes being used (instead of thinking about both subject-knowledge and the processing skill);

**Bridge:** Bridge those processing skills to apply them in a simplified problem situation in a target subject domain; reflect on the process used to solve the simplified problem, or scaffolded problem;

**Extend:** Extend the application of those process skills to any type of problem situation.

Assessment and attitude

We used the McMaster self-assessment format:

pre-test → post-test → Discovery

This helped students become more aware of their attitude, ability, and change throughout the course.

This year we are looking at using a more focused approach to developing problem solving attitude, including the use of a learning log with directed prompts, dyadic interviews, and problem creation/identification exercises.
Building problem solving skills…Step 2: Awareness

Good problem solvers are self-reflective when solving problems. They ask questions of themselves and are conscious of what they are doing, why they are doing it, and where they are going.

One of the most dramatic things I found from the problem solving course is that many students have no awareness at all of what they are doing when approaching a problem.

**Tip:** Use Think-Aloud Pair Problem Solving (TAPPS) to help students develop awareness of their own problem solving approach, and see the approach of other students.
Think-Aloud Pair Problem Solving (TAPPS)

Also known as the Whimbey Pair Method

Basic idea: Verbalise all your thoughts to a listener while trying to solve a problem.

Why?
- we can describe to others "where we are in a problem";
- we can compare "how we do it" with how "others do it";
- we can get ourselves "unblocked" if we cannot seem to solve a problem;
- we need to be able to describe our thoughts for team problem solving;
- we can describe our thought processes to another so that we can improve.

Talking while thinking also helps you to slow down and focus on what you are doing while solving a problem.

Talker: Verbalise everything.
Start by reading the problem aloud. Talk and think at the same time. Try to solve the problem even if you think it is easy.
The job of the Listener

1. Help the talker to see that you are not a "critic". Instead, you are a question-askee. You might say "Please keep talking." or "I was not able to understand or follow what you just said; would you please explain." "Can you tell me what you are thinking now?" "Do not worry about how it sounds- just say an idea about what you are thinking." "Can you check?" "Are you sure?" "OK" "Ahmmm"

2. You might tell the talker that your role is to:
   - remind them to keep talking,
   - help them improve the accuracy of their talking about their thinking,
   - be able to understand and follow each step of the talker's thinking,

3. Do not turn with your back to the talker and try to solve the problem on your own. Do not solve the problem on your own and then tell the talker what they should do.

4. Do not let the talker continue if:
   - you do not understand what they have done,
   - you think that a mistake has been made and therefore they should "check what they have done." If they still do not see what might be wrong, point out but do not correct the error.

Above suggestions taken from the Unit 1 Workshop of the MPS Program:
http://chemeng.mcmaster.ca/mcmaster-problem-solving-mps-program
1. Dracula hates daylight more than Wolfman unless Frankenstein hates daylight more than Dracula. In that case Wolfman hates daylight more than Dracula but less than Mummy. Mummy hates daylight more than Dracula but less than Frankenstein. Show a diagram of the monsters ordered according to their hatred of daylight.

2. The number of cows owned by farmer Smith is the number owned by Farmer Thompson divided by the number owned by farmer Jones. Farmer Thompson, who owns 42 cows, would own 8 times as many cows as farmer Jones if he owned 14 more cows. How many cows does farmer Smith own?
Trying out TAPPS (bridge phase).
Mathematical fallacies, spot the error

1. Let $a$ and $b$ be equal non-zero quantities
   \[ a = b \]
2. Multiply by $a$
   \[ a^2 = ab \]
3. Subtract $b^2$
   \[ a^2 - b^2 = ab - b^2 \]
4. Factor both sides
   \[ (a - b)(a + b) = b(a - b) \]
5. Divide out $(a - b)$
   \[ (a + b) = b \]
6. Observing that $a = b$
   \[ 2b = b \]
7. Divide by the non-zero $b$
   \[ 2 = 1 \]

\[
1 = \sqrt{1} = \sqrt{(-1)(-1)} = i \cdot i = -1
\]
Building problem solving skills…Step 3: Strategy

Good problem solvers have a strategy. They try to understand the problem. They look for connections to problems they know. They try simpler problems. They explore, make mistakes, check their conclusions.

Many students have no coherent strategy. They try to fit formulas to a question. They don’t read the question properly. They don’t use all the information in the question. They make inconsistent leaps. They give up easily.

**Tip:** Introduce a problem solving strategy and focus on the different stages of the problem solving process.

**Tip:** Use a strategy board with TAPPS.
Problem solving strategy

Strategy suggested by G. Polya:
1. Define: Understand the problem
2. Plan: Devise a plan
3. Do it: Carry out the plan
4. Look back: Examine the solution obtained

Strategy as suggested in the MPS program:
Break “Define” phase into three steps...
1. Engage: read the problem, “I want to & I can”
2. Define the stated problem
3. Explore: Create internal idea of problem
4. Plan a solution
5. Do it: Carry out the plan
6. Evaluate: Check & Look Back
Where to focus attention? Where do students tend to struggle?

At which stage of the problem solving process do you find students tend to struggle?

1. Engage phase: wanting to solve the problem
2. Define phase: Understanding the problem
3. Explore phase: Creating an internal representation of the problem
4. Planning phase: Creating a plan to solve the problem
5. Execution phase: Carrying out the plan
6. Looking back phase: Checking the solution is correct and reflecting on what has been learned while solving the problem
7. Students have equal difficulty with all the phases
8. None of the above
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Students tend to struggle in the Define phases. They don’t read the question properly, skim over things they don’t understand, and try to rapidly move to the “Do It” phase.
TAPPS with the strategy board

**Talker** now also moves a marker, identifying which stage of the problem solving process they are up to.

**Listener** keeps track of the time at each stage. Does not move the marker. Might simply ask a question like “Are you still at the Define phase?” if it looks like the talker has moved on, but not moved the marker.
Trying out TAPPS with the strategy board

   Subject: “915.”
   Magician: “Then the number you started with is ....”
   How can the magician quickly find the number that the subject originally selected? Explain the trick.

2. When the tortoise raced the hare, the former maintained a constant pace of one kilometre per hour throughout the race, while the latter, being overconfident, wasted much time and averaged only half a kilometre per hour for the first half of the course. How fast must the hare run over the second half of the course in order to win?

3. Sue Ling is three times as old as Chin Lee was when Sue was as old as Chin is now. When Chin is as old as Sue is now, Sue will be 56. How old are Sue and Chin now?

4. (Mahavira) One-fourth of a herd of camels was seen in the forest; twice the square root of that herd had gone to the mountain slopes; three times five camels remained on the river bank. What is the numerical measure of that herd of camels?
Building problem solving skills…Step 4: Self-reflection.

**Tip:** Try a journal. Or if you don’t like that name, perhaps call it a “problem solving notebook”.

Give students some time in a lecture to jot down insights which might have occurred to them when solving a particular problem; connections between material they might have made; or self-assessments of their own abilities (where they had trouble, or if there is an area of knowledge which they discovered they need to check up on).

Try looking at it occasionally. It doesn’t have to be assessed.

I also give students **pre-** and **post-tests** in which they rate their own ability in particular skills or areas. These I also ask to be included in the journals.
Some general tips for setting questions

• Setting problems as questions is harder than setting exercises.
• Problem solving is hard, so start easy:
  • start with questions that are just a little bit unusual, where the student has to make a small original contribution. Perhaps comparing special cases, or inferring something from a graph.
  • Take an exercise, and remove any reference to a prescribed method
  • Build towards harder problems with heuristics specified (e.g. “Draw a diagram” or “Solve a simpler but related problem”)
  • Ask students to come up with their own problems
• Assessment is more challenging as the level for problems is hard to gauge. Some students may solve the problems easily, others may spend a long time on a problem.
  • Focus on the process, so students who are having difficulties can still show their skills.
• Be prepared to offer support. I use a forum. Recently I have started to use Piazza and the students are much more engaged with this (than the inbuilt Moodle forum).
Summary

To improve student problem solving skills we need to teach the skills, and encourage their use.

Many questions we ask students are exercises, not problems.

Student attitude is particularly important for problem solving success.

We can use the TAPPS method to increase problem solving awareness (note, TAPPS serves a different purpose to the peer-instruction method, and so doesn’t replace peer-instruction).

We can use a Strategy Board, and TAPPS, to make the use of a problem solving strategy explicit.

We can use a journal to encourage students to reflect on their learning.
Resources

1. “Thinking, fast and slow”, D. Kahneman
2. “How to think like a mathematician”, K. Houston
3. “Student engagement techniques: A handbook for college faculty”, E.F. Barkley
5. “Teaching at its best”, L.B. Nilson
6. “How to solve it”, G. Polya
7. “53 Ways to Ask Questions in Mathematics and Statistics”, R. Hubbard
10.“Puzzle-based learning: An introduction to critical thinking, mathematics and problem solving”, Z. Michalewicz and M. Michalewicz
11.McMaster Problem Solving course
   [http://chemeng.mcmaster.ca/sites/default/files/media/mps2.pdf](http://chemeng.mcmaster.ca/sites/default/files/media/mps2.pdf)
12.“My Best Mathematical and Logic Puzzles”, M. Gardner
13.“Problem Solving & Comprehension”, A. Whimbey, J. Lochhead and R. Narode
   (TAPPS is described in Ref. 13 and on the McMaster site (not the pdf above))