Designing and using interactive applets for conceptual understanding

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Visualisation and conceptual thinking

How can we support our students to develop a solid conceptual understanding of mathematics & statistics?

- visualisation

  visual representations of concepts, relationships

  abiding images

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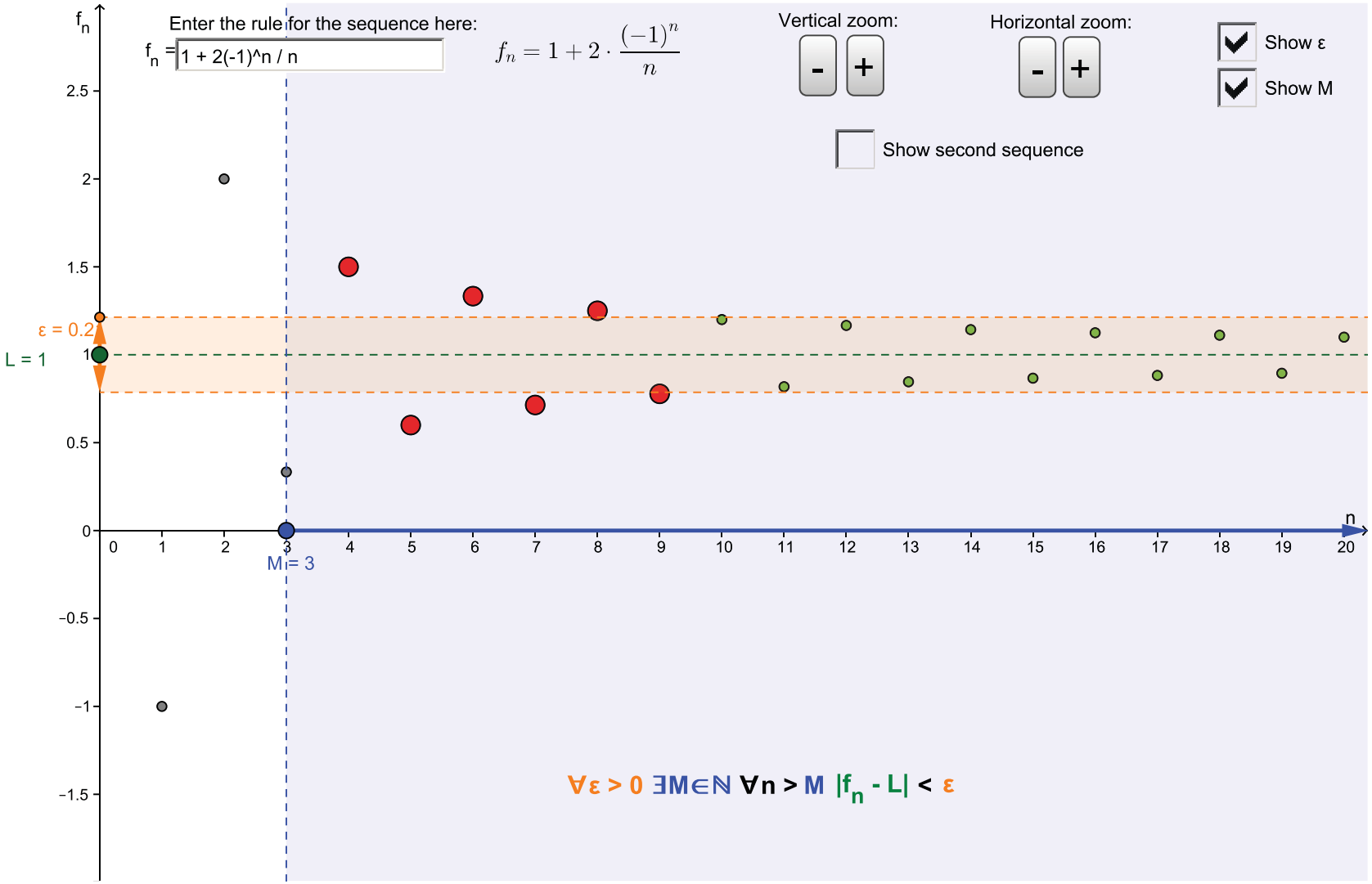
$\longrightarrow$ interactive applets for conceptual learning

Interactive applets

Why use applets?

- Visual representations of concepts, relationships
- Targeted conceptual focus
- Tailored to a particular teaching context
- Transferrable across learning/teaching domains
- Flexible – multiple uses, entry points
- Accessible – low barriers to use
- Interactive – telling a story
- Engaging – fun, creative thinking

Limit of a sequence - ϵ-M



Limit of a sequence - ϵ-M

Visualisation:

- blue/orange regions
- red/green points

Targeted:

- Difficult but important concept
- Compare two sequences – based on teaching need

Flexible:

- convergence
- divergence
- bounding

Limit of a sequence - ϵ-M

Coherence:

- Same notation as lectures
- Same colour/layout as related ϵ-δ applet

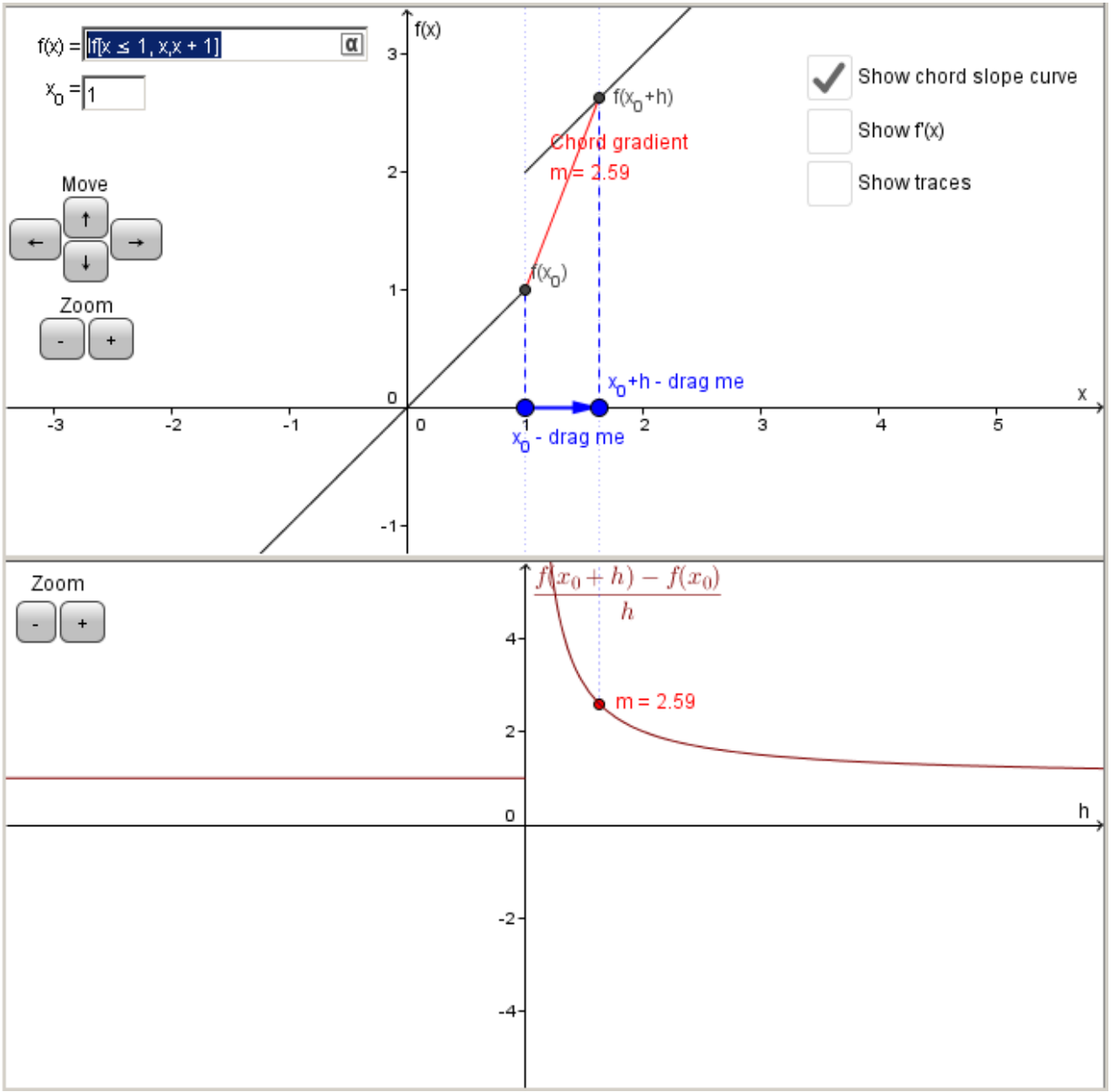
Transferrable:

- Use in lectures, one-on-one consultations
- Common 'visual vocabulary' for discussions

Interactive:

- Reveal components one-by-one when ready
- Enhances dialogue

Differentiability



Differentiability

Visualisation:

- Multiple representations
- Clear image of why/how differentiability fails

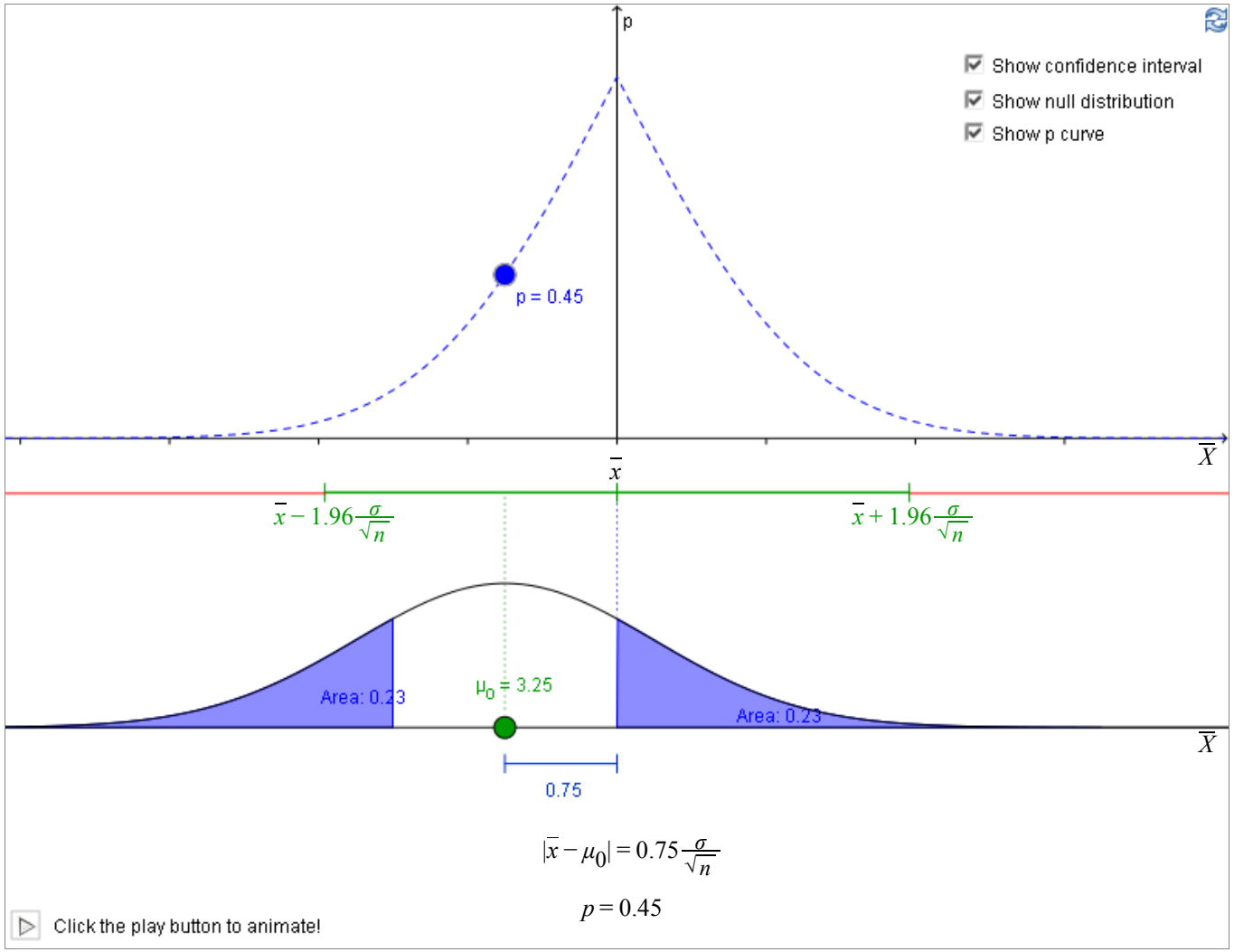
Targeted:

- Deep understanding of concept
- Address common misconceptions
- Supports key examples

Interactive:

- Leaves a ‘trace’ of previous actions

CI's, hypothesis testing and p-values



CI's, hypothesis testing and p-values

Visualisation:

- Linking concepts often treated separately
- Multiple visual representations of accept/reject regions
- Challenging viewpoint: $\bar{x}$ is fixed, μ_0 changes

Flexible:

- Simple: accept/reject regions and confidence interval
- More challenging: p vs. μ_0

CI's, hypothesis testing and p-values

Interactive:

- Question: what would the graph of p vs μ_0 look like?
- Think then test
- Reveal components one-by-one when ready

Engaging:

- 'Drag me!'

Principles

Minimise technological barriers

- applet ‘just works’ in most browsers, devices
- uses familiar syntax
- hosting taken care of by Geogebratube
- easily distributed via web link, etc

Principles

Minimise cognitive load

- correspondence between user interface elements (view) and conceptual elements (model)
- physical interaction - tactile, 'embodied cognition'
- colour coding of semantically related elements

Reduce extraneous mental effort

Maximise mental resources available for concepts

What else are we doing?

Applets for

- Calculus: sequences & series, Riemann sums, ODEs
- Statistics: confidence intervals & hypothesis testing, power, random variables, order stats, MLEs, ...
- Others: eigenvectors, difference equations

What else are we doing?

Supporting resources

- online tutorial exercises
- teaching notes
- ‘how-to’ guides or similar

Evaulation

- quick surveys immediately after applet use
- collect analytics data
- focus groups, interviews etc

GeoGebra

The applets are constructed in GeoGebra

- Freely available interactive geometry/graphing/CAS system
- Open source
- Java application, cross-platform (Windows, Mac, iPad ...)
- Developed by educators, for education
- Increasingly popular in secondary education

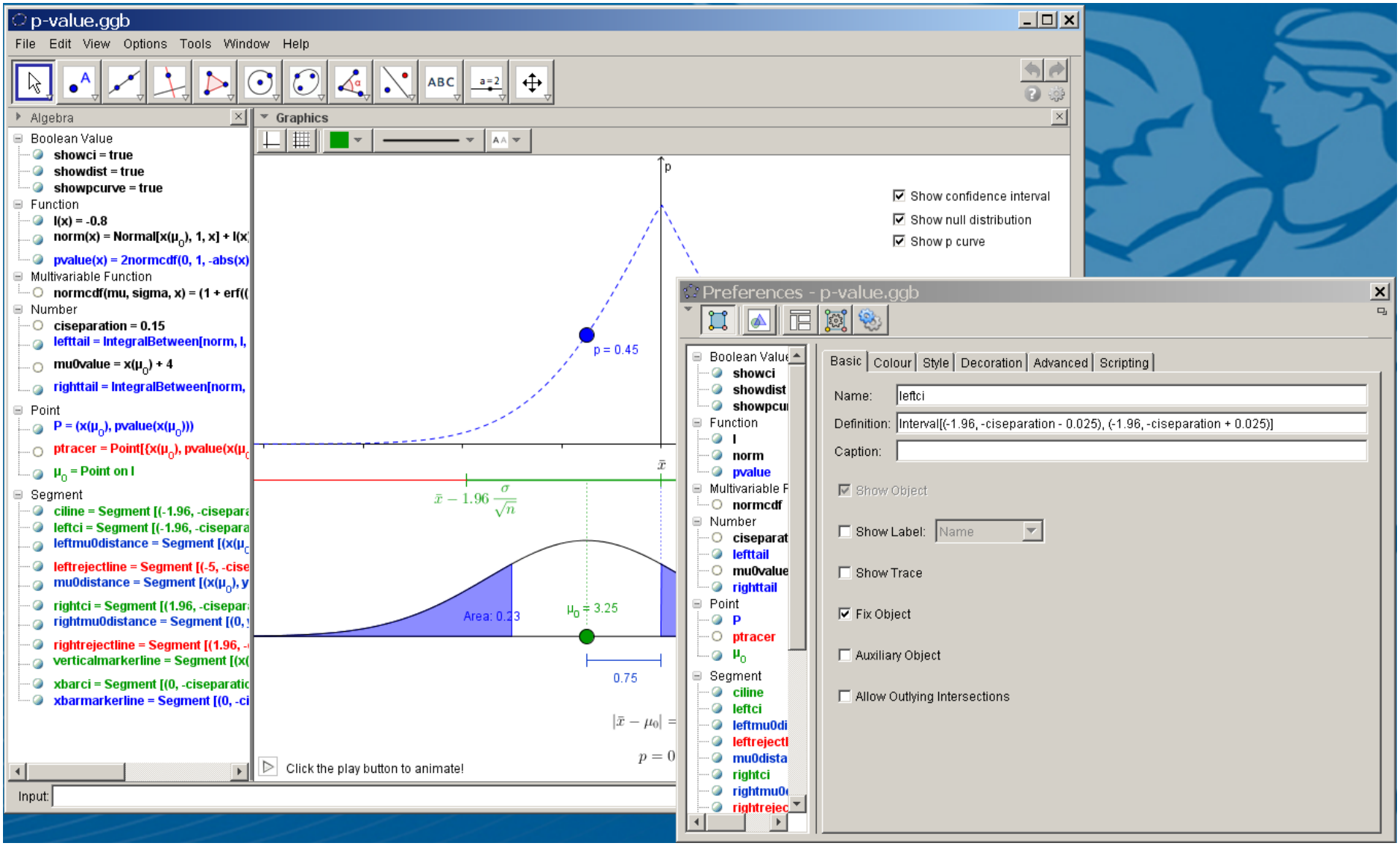
www.geogebra.org

GeoGebra

Geogebra is a good platform for such projects.

- Rapid development
- Minimal programming - build by construction
- Extensive documentation & community support
- Exports to HTML5 - no Java, plugins required!
- Host applets publicly (Geogebratube) or privately (Moodle, etc)

GeoGebra


p-value.ggb
File Edit View Options Tools Window Help
Algebra
Graphics
Boolean Value
showci = true
showdist = true
showpcurve = true
Function
l(x) = -0.8
norm(x) = Normal[x(μ0), 1, x] + l(x)
pvalue(x) = 2normcdf(0, 1, -abs(x)
Multivariable Function
normcdf(mu, sigma, x) = (1 + erf((
Number
ciseparation = 0.15
lefttail = IntegralBetween[norm, l,
mu0value = x(μ0) + 4
righttail = IntegralBetween[norm,
Point
P = (x(μ0), pvalue(x(μ0)))
ptracer = Point[{x(μ0), pvalue(x(μ0
μ0 = Point on l
Segment
ciline = Segment [(-1.96, -cisepara
leftci = Segment [(-1.96, -cisepara
leftmu0distance = Segment [(x(μ0
leftrejectline = Segment [(-5, -cise
mu0distance = Segment [(x(μ0), y
rightci = Segment [(1.96, -cisepara
rightmu0distance = Segment [(0,
rightrejectline = Segment [(1.96,
verticalmarkerline = Segment [(x(
xbarci = Segment [(0, -ciseparatio
xbarmarkerline = Segment [(0, -ci
Input:
p
Show confidence interval
Show null distribution
Show p curve
p = 0.45
$\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}$
$\bar{x}$
Area: 0.23
$\mu_0 = 3.25$
0.75
$|\bar{x} - \mu_0| =$
$p = 0$
Click the play button to animate!
Preferences - p-value.ggb
Boolean Value
showci
showdist
showpcur
Function
l
norm
pvalue
Multivariable F
normcdf
Number
ciseparat
lefttail
mu0value
righttail
Point
P
ptracer
μ0
Segment
ciline
leftci
leftmu0di
leftrejectl
mu0dista
rightci
rightmu0
rightrejec
Basic Colour Style Decoration Advanced Scripting
Name: leftci
Definition: Interval[(-1.96, -ciseparation - 0.025), (-1.96, -ciseparation + 0.025)]
Caption:
Show Object
Show Label: Name
Show Trace
Fix Object
Auxiliary Object
Allow Outlying Intersections

Our applets may be found at

http://www.melbapplets.ms.unimelb.edu.au

or at our GeoGebratube profile

http://geogebratube.org/user/profile/id/36916

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