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Undergraduate mathematics students tripped by pronumeral misconceptions

Caroline Bardini MGSE

Robyn Pierce, Jill Vincent MGSE

Deborah King Department of Mathematics and Statistics

Background

Incorrect or incomplete mathematical conceptions persist unnoticed

- From primary school
- From secondary school

Unless noticed and addressed they do not magically disappear.

This study

Our conjectures:

- Some university mathematics students may struggle because their mathematical thinking includes fundamental misconceptions.

- Students may demonstrate success in senior secondary school mathematics through practice of mathematical routines and familiarisation with standard examination questions while still harbouring fundamental misconceptions.

Focus of the study

Algebra

A known source of difficulty
The subject of much research

Misconceptions regarding the meaning of **letters** and understanding of **variables**

Functions

Previously discussed (Bardini, Pierce & Vincent, 2013)

Much of the focus has been on junior secondary mathematics and the early transition to algebra. What remains at the end of the chain (and beginning of another)?

The case of pronumerals

Based on research literature we conjecture that students may:

- still believe that pronumerals represent objects, labels or abbreviations
- have difficulty developing problem formulation skills, since they do not fully understand the role of pronumeral assignment (variable vs. unknown)
- assume that the domain of all pronumerals is the set of real numbers (rather than set needs to be specified in each instance).

Misconceptions come into light when questions are given in an unusual form

Our study -Methodology

1st year maths and stats students soon after the start of their first semester at UoM.

- Background demographic survey
- Mathematics quiz (not a formal part of any subject)
  - Accessed through the university's Learning Management System
  - Time limited (35 min) & one attempt only
- 'Think-aloud' interviews for small number of students

The Mathematics Quiz

16 questions designed to probe understanding of both pronumerals and functions.

The responses of the 383 students who attempted the quiz (427) *and* answered the survey (>600) were analysed.

Quiz question 13

When is the equation $L+M+N = L+P+N$ true?

(a) Always

(b) Never

(c) Sometimes

Used in past research (junior high school and university)

Designed to further investigate findings (e.g. Stephens 2005) that indicate students believe that "when a letter represents a number, **usually each letter represents a different number**".

Results q.13

$L+M+N = L+P+N$ true?

	% students (n = 383)	% respondents (n = 337)
Always	1.8	2.1
Never	**20.4**	**23.1**
Sometimes	65.8	74.8

Quiz question 15

Some students were asked to find values of *x* that would make the following equation true: $x + x + x = 12$

Select each student whose answer is correct (choose as many as apply).

(a) Mary wrote x=2, x=5 *and* x=*5*

(b) Millie wrote x=9, x=2 *and* x=*1*

(c) Mandy wrote x=4

Designed to test students' belief "when a letter represents a number, usually **each letter represents a different number**"

Question 15 results

failed to understand the role of x in the equation , that is while x stands for a variable, within the one equation each separate appearance of x must stand for the ***same*** number

	% students (n=383)	% respondents (n = 334)
a, b & c all correct	8.6	9.9
a & b only	1.3	1.5
a & c only	0.3	0.3
a only (2,5,5)	0.8	0.9
b only (9,2,1)	0.3	0.3
c only (4)	76.0	87.1

$$x + x + x = 12$$

Comments q. 15

- Had these students been presented with the equation in the form '3x=12' or if they had been asked to simplify the equation first, very likely they would all have recognised that 'x=4' was the only correct answer.

- When students have been used to practising repetitive equation solving in their early algebra experiences, presenting them with a non-routine problem highlights the basic misconceptions which many students manage to carry with them

Quiz question 16

Some students were asked to find values of *x* and *y* that would make the following equation true: $x + y = 16$

Select each student whose answer is correct (Choose as many as apply).

(a) John wrote $x=6$ and $y=10$

(b) Jack wrote $x=8$ and $y=8$

(c) James wrote $x=9$ and $y=7$

Designed to test students' belief "when a letter represents a number, usually **each letter represents a different number**"

Results q. 16

	% students (n = 383)	% respondents (n = 333)
a, b & c all correct	65.5	75.4
a & b only	0.3	0.3
a & c only	**18.8**	**21.6**
a only (6,10)	1.0	1.2
b only (8,8)	1.3	0.5
c only (9,7)	0	0

$x + y = 16$

Conclusions

Despite difference in schooling systems and decades that separate our study from previous studies

- A significant number of first semester undergraduate mathematics students in this study held similar misconceptions relating to understanding of the use of pronumerals
- Proportion of students with these misconceptions in our study is not large
- Nevertheless of considerable concern
- Students will start to study relationships between variables (functions) and how the variables change in relation to each other (calculus). Hence a deep understanding of different types of pronumerals and how to define them is vital.

Conclusions (cont')

- These are students who have passed very successfully through the school system with sufficiently high scores in their final year of school to be accepted into first year mathematics at a high profile university.

- Reasonable to hypothesize that across the country, the proportion of students with these misconceptions would be much higher.

Implications

- **Tutors need to be alert to incorrect patterns of thinking – not just one-off slips**
- Making diagnostic testing available is important
- Vital to use carefully constructed non-routine questions to uncover students' incorrect or incomplete conceptual understanding of mathematical ideas
- Suitable, research informed tests are now readily available 'online' to teachers and their students, see for example Specific Mathematical Assessments that Reveal Thinking (www.smartvic.com)

Implications (cont')

At all levels we need to

- Be explicit about the meaning and role of symbols
- Promote mathematical thinking and sense making
- Balance symbol manipulation practice with problem-solving experiences so that these different roles of pronumerals have meaning for students

Next steps

ARC DP 150103315 (2015-2017) - **Secondary and university mathematics: do they speak the same language?**

- Seeks innovative reasons for low progression rates of students in STEM subjects in Australia
- Examines students' symbol use in mathematical sciences at university vs. use at school
- Links between students' response to increased symbolic load and their confidence to continue studying subjects with high mathematical content at university

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Any questions?

Contact: Caroline Bardini (cbardini@unimelb.edu.au)

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